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Controlling light by light in a one-dimensional resonant photonic crystal

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We consider the interaction and stability of gap solitons in a one-dimensional, resonant, photonic crystal with a defect state produced by a linear localized mode, or by an incoherent pump. Soliton propagation is investigated using both an analytical treatment and the direct numerical integration of the two-wave Maxwell-Bloch equations. Our results demonstrate that the soliton can be trapped, reflected from, or tunnel through the defect state.

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The high level of current interest in light dynamics within periodic structures, or photonic crystals (PCs), is due to a combination of improvements in fabrication and the novel physics associated with such nonlinear periodic structures [1–4]. The periodic variation of the material properties results in a forbidden band gap, in which light cannot propagate in the linear regime. By including a resonant [5], cubic [6], or quadratic [7] nonlinear material within the structure, it is possible to realize a situation where a slow, gap soliton can propagate through the stop band. In such structures, coupling of the two Bloch waves which are created by the periodic nature of the material produces a slow gap soliton (GS) whose amplitude and velocity experience periodically change [8–10]. In this paper, we consider resonant PCs where the nonlinearity is due to the retarded coherent response of resonantly absorbing material. We develop an analytical model to describe the propagation of gap solitons and their interaction with a localized linear mode or defect state.

The properties of resonant photonic crystals (RPCs) can be modeled using a Maxwell-Bloch system of coupled partial differential equations. This formalism has been studied in Ref. [10], where a Langrangian was derived and a relevant center-of-mass equation was found. In turn, this enabled them to find the bound state of the *perturbed* GS and the localized linear wave of the RPC. This type of interaction may become important in the case of the *exact* GS if its propagation is accomplished by an excitation of a weak linear mode of the RPC that may be caused by a residual localized resonant field, created, for example, by an incoherent pump, or periodicity breakup. Indeed, it has already been shown that the resonant GS can be trapped by a steady-state structural defect [11] or a region of localized gain [12,13].

In this paper, the interaction between the exact GS and a localized linear mode in a one-dimensional (1D) resonant, photonic crystal is investigated using both an analytical treatment and numerical integration of the two-wave Maxwell-Bloch equations. The interaction between the solitons and the defect state is shown to modify the GS dynamics, resulting in trapping, reflection, and deflection of the soliton path. The same effects are also observed when the GS traverses a defect state composed of resonant centers, atoms, ions, or excitons which are incoherently inverted. Therefore, we demonstrate the possibility of controlling an intense GS by means of a weak linear field or by a low-intensity incoherent pump, that is, without creating a defect.

The propagation of a scalar field through a 1D PRC, which consists of a periodic sequence of alternating thin layers containing two-level centers with a period equal to the resonant wavelength, is governed by the following set of the normalized Maxwell-Bloch equations [8,10]:

$$\Omega_t + \tilde{\Omega}_x = -2\sin\theta, \tag{1a}$$

$$\tilde{\Omega}_t + \Omega_x = 0, \tag{1b}$$

$$\theta_t = \Omega, \tag{1c}$$

where $\Omega=\Omega^+ + \Omega^-$, $\tilde{\Omega}=\Omega^+ - \Omega^-$, $P(x,t)=-\sin\theta(x,t)$ is the dimensionless resonant polarization, $n(x,t)=-\cos\theta(x,t)$ is the inversion population, $\Omega^\pm$ are the slow amplitudes of the forward and backward waves, $\theta(x,t)$ is the Bloch angle, t and x are the dimensional time and propagation coordinate, respectively, and the subscript refers to the corresponding derivative.

An invariable quantity $\Gamma(x)$ associated with this set can be obtained after some manipulations from Eqs. (1b) and (1c),

$$\tilde{\Omega}(x,t) + \theta_x(x,t) = \Gamma(x). \tag{2}$$

This conservation parameter is defined by the initial conditions and does not change as the light pulse evolves inside the RPC structure. However, it does dictate the behavior of the GS in the sense that setting it equal to zero provides a GS, whereas a nonzero value of $\Gamma(x)$ leads to a GS instability and trapping inside the RPC. It is clearly seen from the relevant substitution that transforms Eq. (1a) into the perturbed sine-Gordon equation for the Bloch angle θ,

$$\theta_{xx}(x,t) - \theta_{tt}(x,t) = 2\sin\theta(x,t) + \Gamma_x(x), \tag{3}$$

where the perturbation is now defined by the conservation parameter $\Gamma(x)$.

In order to elucidate the interaction between the exact GS and localized linear modes, we apply the approach developed earlier in Ref. [10], following which the coordinate of the GS

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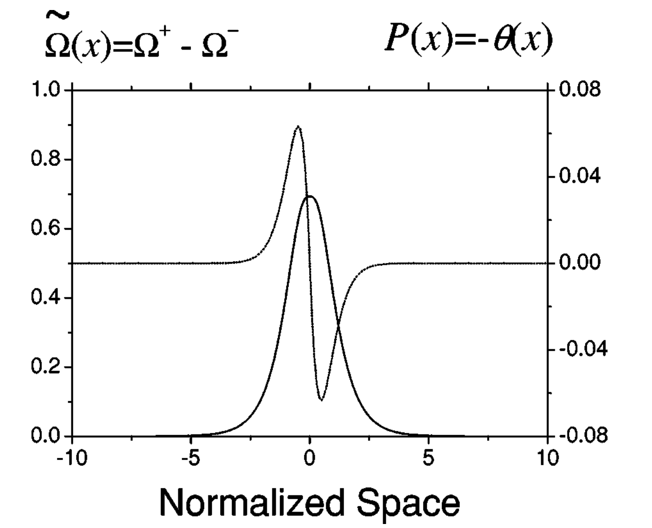

FIG. 1. The difference field $\tilde{\Omega}(x)=\Omega^+(x)-\Omega^-(x)$ (solid line) and polarization $P(x)=-\theta^{(l)}(x)$ (dotted line), corresponding to the linear mode at $\Gamma_0=1$.

center-of-mass $\xi(t)$ can be defined from a classical equation of motion of the particle with unit mass, $\xi_{tt}=-\Phi_\xi$, with an effective potential of the interaction $\Phi(t)=1/4\int_{-\infty}^{+\infty}dx\Gamma_x\theta_x^{(l)}$, where $\theta^{(l)}$ is the linear localized mode defined as a solution of the steady-state linear equation,

$$\theta_{xx}(x,t) = 2\ \theta(x,t) + \Gamma_x(x). \tag{4}$$

Now assume that the conservation value has the form $\Gamma(x)=\Gamma_0\ \mathrm{sech}(\sqrt{2}\ x)$, the physical meaning of which is an appearance of the nonzero difference field $\tilde{\Omega}$ which, in turn, generates a low-intensity localized perturbation. Making use of this conservation quantity along with assuming $\theta^{(l)}\propto\exp(-\sqrt{2}|x|)$ gives the localized solution of the perturbed Eq. (4)

$$\theta^{(l)}(x) = \frac{\Gamma_0}{\sqrt{2}}\{\sqrt{2}x\ \cosh(\sqrt{2}x) - \sinh(\sqrt{2}x)\ln[2\ \cosh\sqrt{2}x]\}. \tag{5}$$

Subsequently, the localized linear Bloch waves $\Omega^\pm(x,t)$ and medium polarization $P(x,t)$ can now be defined by the invariant (2) and condition $\Omega=0$ as follows:

$$\pm\Omega^\pm(x) = \tilde{\Omega}(x)/2, \tag{6a}$$

$$\tilde{\Omega}(x) = \Gamma_0\ \mathrm{sech}(\sqrt{2}x) - \theta_x^{(l)}(x), \tag{6b}$$

$$P(x) = -\ \theta^{(l)}(x). \tag{6c}$$

These solutions are presented in Fig. 1, where the relevant comparison says that the resonant polarization $P(x)$ and Bloch-angle derivative $\theta^{(l)}/x$ are much smaller than the difference $\tilde{\Omega}$ between the forward- and backward-propagating waves. This means that the perturbation term in Eq. (3) is mainly driven by this difference component. This is consistent with the physical meaning of the linear localized mode.

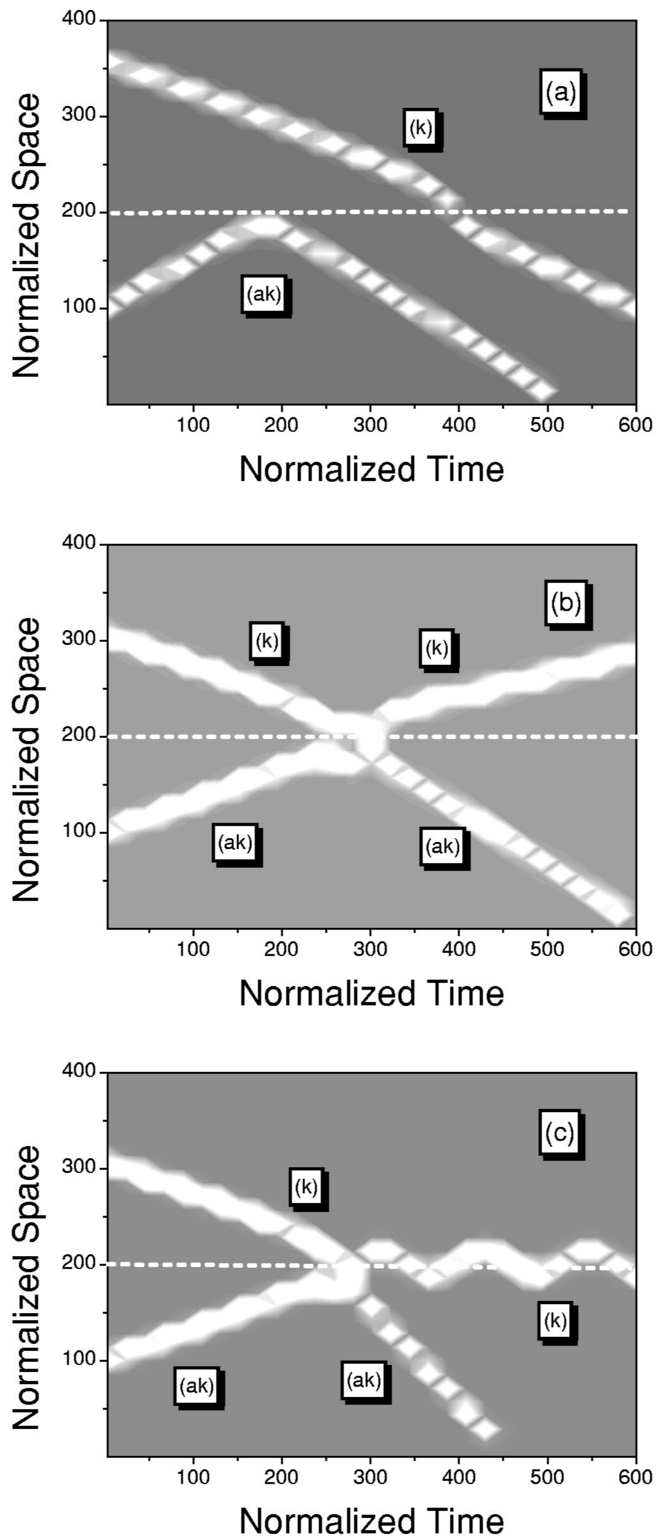

FIG. 2. Dynamics of kink (k) and antikink (ak) interaction with the localized linear mode as an inversion projection onto the (x,t) plane; $\Gamma(x)=\Gamma_0\ \mathrm{sech}[\sqrt{2}(x-x_0)], x_0=200, u=0.03$. Notice the difference between single-pulse elastic scattering at the localized linear mode with $\Gamma_0=0.015$ (a) and inelastic kink-antikink collision at localized linear modes with $\Gamma_0=0.01$ (b) and $\Gamma_0=0.012$ (c), correspondingly.

It is a steady-state optical field localized on resonant centers which are slightly excited above the ground state such that $P(x) << 1$ and $n(x) \approx -1$.

In application to the problem of the interaction of the exact GS with such a localized mode (5) and (6), it allows us to rewrite the interaction potential as

$$\Phi(t) = \frac{\Gamma_0}{4}\int_{-\infty}^{+\infty} \mathrm{sech}(\sqrt{2}x)\tilde{\Omega}(x,t)dx. \qquad (7)$$

If the GS is represented by the antikink solution of the sin-Gordon equation,

$$\theta(x,t) = 4 \arctan\left[\exp\left(\frac{\sqrt{2}[-x+\xi(t)]}{\sqrt{1-u^2(t)}}\right)\right], \qquad (8)$$

with $u(t)=\xi_t$ being the velocity of the soliton, the field $\tilde{\Omega}$ is equal to

$$\tilde{\Omega}(x,t) = \frac{2\sqrt{2}}{\sqrt{1-u^2}}\mathrm{sech}\left[\frac{\sqrt{2}(x-\xi)}{\sqrt{1-u^2}}\right], \qquad (9)$$

and can be large enough for the potential (7) to exceed the kinetic energy, $u^2/2$, of the slow GS. When $\Gamma > 0$, the linear mode of the RPC reflects the GS with $\tilde{\Omega} > 0$ (antikink) and captures that one with $\tilde{\Omega} < 0$ (kink), which is shown in Fig. 2(a) When $\Gamma \neq 0$, the two-wave Maxwell-Bloch equations are nonintegrable and are reduced to the perturbed sine-Gordon equation; the combined action of the slow velocity of the GS soliton propagation and perturbation makes the soliton interaction inelastic [14]. The kink GS is reflected by the attractive potential (9) at the collision with the antikink [Fig. 2(b)], but a subtle increase of the Γ_0 leads to the kink trapping and an oscillating GS appears [Fig. 2(c)] that is the bound state of the GS and linear mode [10].

It is straightforward to assume that the dynamics of slow GSs may also be controlled by means of their interaction with other perturbations. As a relevant example, Fig. 3 shows the results of the numerical integration of the Maxwell-Bloch equations (1) for the case of the GS interaction within a length of incoherently excited resonant atoms with

$$n(x,t) = -1 + v\ \mathrm{sech}[\sqrt{2}(x-x_0)]. \qquad (10)$$

A significant feature here is again a kink trapping in the form of the oscillating GS that follows its collision with the antikink GS and the escape of the latter from the interaction area; a considerable change of the velocity of the antikink GS is also noticeable.

In conclusion, we have demonstrated that by using an analytical method, interesting and previously unforeseen properties of GSs in RPCs can be predicted and explained in a physically transparent form. The most important result is the fact that we are able to show that the oscillating GS created because of the linear perturbation can be described, at least qualitatively, as an effective-potential-like effect. Using this method, we were able to obtain qualitative agreement with direct numerical solution. Future work on GS dynamics can be directed toward an examination of new features that may be brought about by the implementation of our findings on, say, a semiconductor device format. Recent experiments reported on linear and nonlinear light dynamics in $In_xGa_{1-x}As/GaAs$ periodic multiple-quantum-well structures [15,16], and success in the manufacturing of Er-doped $Al_xGa_{1-x}As/GaAs$ nanostructures [17] suggests that the behavior predicted in this paper might be experimentally feasible at light intensity $\sim$10 MW/cm^2. This value is consistent with the experimental observation of gap solitons in $AI_xGa_{1-x}As$ using the half-band-gap nonlinearity where intensities of $\sim$1GW/cm^2 were used [3] and recent predictions of gap solitons in coupled microring resonators with intensities of $\sim$50 MW/cm^2 have been estimated. The use of a resonant nonlinear system should result in optical intensity levels which can be easily attained using current laser technology. The use of a resonant photonic crystal to investigate the dynamics of gap solitons will lead to a rich and complex physical system which promises to generate novel physical results.

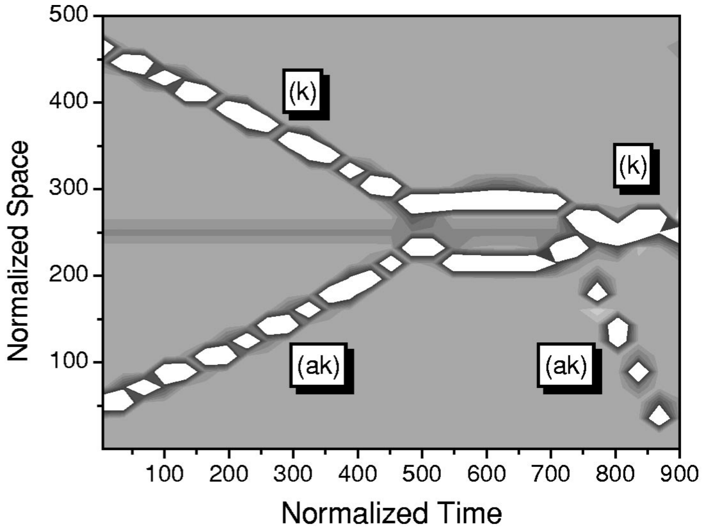

FIG. 3. The inversion projection $n(x,t)$ for kink (k) and antikink (ak) interaction on the incoherent inversion length with $v=0.11$ and $x_0=250$.

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