JETP Letters, Vol. 82, No. 5, 2005, pp. 253–258. Translated from Pis'ma v Zhurnal Éksperimental'noĭ i Teoreticheskoĭ Fiziki, Vol. 82, No. 5, 2005, pp. 284–289.

Optical Zoomeron as a Result of Beatings of the Internal Modes of a Bragg Soliton

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Received June 22, 2005; in final form, July 12, 2005

A new solution of two-wave Maxwell–Bloch equations has been obtained analytically and numerically. It describes the propagation of an oscillating nonlinear optical solitary wave, or optical zoomeron, in a one-dimensional periodic resonant Bragg structure. It has been shown that the appearance of large oscillations in the velocity and total amplitude of Bloch modes of the pulse is caused by beating of internal modes of the perturbed Bragg soliton. © *2005 Pleiades Publishing, Inc.*

PACS numbers: 42.25.Fx, 42.50.Md, 42.65.Tg, 42.70.Qs

Investigations of the dynamics of nonlinear wave processes involving solitary nonlinear waves, or solitons, are continuously attracted great interest in various areas of natural sciences and engineering [1]. First of all, this interest is associated with a rich variety of nonlinear dynamic systems in nature whose evolution is in high extent determined by the unique properties of solitons, namely, the conservation of the shape and velocity during propagation and after interaction. Strictly speaking, such properties are characteristic of only solutions of completely integrable nonlinear dynamic equations such as the sine-Gordon equation, Maxwell–Bloch equations, nonlinear Schrödinger equation, etc., which appear as a result of the use of certain approximations when solving a number of physical problems. Using the inverse-scattering method, one can mathematically construct an infinite number of completely integrable equations including those that, being taken with various initial conditions, can have both traditional soliton solutions and qualitatively new solutions called zoomerons [2]. A zoomeron is stable when propagating and interacting that is typical for a soliton, but it exhibits new dynamics such that its amplitude and velocity oscillate considerably during motion, and a change not only in the magnitude, but also in the sign of the velocity of the pulse is possible. For this reason, the appearance of zoomeron-like equations in actual physical problems would provide rich possibilities for studying new dynamic laws for nonlinear systems and would allow the generalization of various results concerning soliton dynamics to the case of oscillating pulses. Unfortunately, a physical phenomenon that is described by the completely integrable zoomeron equation has not yet found. At the same time, it is known [3] that a change in the magnitude of the velocity of soliton-like solutions in equations close to completely integrable is possible, for example, in the trapping of the soliton by perturbation, when oscillations with zero average velocity arise near the perturbation, as well as in the inelastic collision of pulses that is accompanied by the single excitation and absorption of the internal mode of the soliton [4]. Long-lived oscillations of the soliton amplitude are also possible when the internal mode is excited at nonzero frequency [5], but the velocity of the soliton is conserved or changes insignificantly in this case. The Bragg solitons of incompletely integrable Maxwell–Bloch equations [6] and nonlinear Schrödinger equations for coupled modes [7] are characterized by dynamic multistability, when, under certain initial conditions, oscillations arise in the velocity of the pulse with the characteristic change of the sign of the velocity, but only for zero average value. Numerical simulation of the dynamics of Bragg solitons in a resonantly absorbed lattice in the case of small detuning from the exact Bragg condition reveals strong oscillations of the amplitudes of Bloch waves and velocity of the Bragg soliton propagating with nonzero average velocity [8]. However, a physical cause of the appearance of such pulse dynamics has not yet been understood.

In this work, the problem concerning the excitation of the internal mode in the standing Bragg soliton of the self-induced transparency with perturbed envelops of the direct and inverse Bloch waves is solved. It is shown that two internal modes close in shape can be simultaneously excited at low and zero frequencies. As a result of beatings of these modes, a periodic energy exchange arises between the internal-mode fields and the resonant subsystem of two-level atoms in the Bragg soliton, which results in the appearance of oscillations in the inversion of excited atoms in the Bragg soliton. The solution is generalized to the case of a slowly moving soliton. Such a soliton is already perturbed due not only to the profile deformation, but also to inversion oscillations accompanying the internal-mode beatings, which results in strong oscillations of the amplitude, polarization, inversion, and velocity of the pulse. Such a dynamics of a solitary wave is characteristic of a zoomeron. The parameters of the solutions obtained by the direct numerical integration of two-wave Maxwell–Bloch equations agree well with the proposed analytical solution for the optical zoomeron-like pulse. The time dependence of the zoomeron velocity is obtained using the energy integral.

The problem concerning the coherent interaction of laser radiation with a one-dimensional resonant Bragg structure of periodically located thin layers containing two-level oscillators is described by two-wave Maxwell–Bloch equations [9] for the slow complex amplitudes of the electric field $E^{\pm}$ of the direct and inverse Bloch waves, average atomic dipole moment P normalized to the transition dipole moment, and inversion @n:

$$\Omega_t + \tilde{\Omega}_x = 2P, \quad \tilde{\Omega}_t + \Omega_x = 0, \tag{1.1}$$

$$P_t = n\Omega, \quad n_t = -\frac{1}{2}(P^*\Omega + P\Omega^*). \tag{1.2}$$

Here, $\Omega = \Omega^+ + \Omega^-$; $\tilde{\Omega} = \Omega^+ - \Omega^-$; $\Omega^{\pm} = (2\tau_c\mu/\hbar)E^{\pm}$; τ_c is the cooperative time; μ is the matrix element of the transition density matrix; $x = x'/c\tau_c$ and $t = t'/\tau_c$ are dimensionless variables, where x' and t' are the space coordinate and time, respectively, and c is the speed of light; and the subscripts x and t stand for the respective partial derivatives. Equations (1) are written under the exact Bragg condition and for the identical frequencies of radiation and resonance transition of oscillators.

We first obtain an expression for the internal mode of the perturbed Bragg soliton with zero propagation velocity and then generalize the solutions to the case of a slowly moving soliton.

Equations (1) have the following integrals of motion corresponding to the conservation of the total energy W and topological charge Q of the localized $\Omega(x = +\infty; t) = 0$ solution:

$$W = \int_{-\infty}^{\infty} \frac{1}{4}\Omega\Omega^* + \frac{1}{4}\tilde{\Omega}\tilde{\Omega}^* + (1+n)dx,$$
$$Q = \int_{-\infty}^{\infty} \tilde{\Omega}dx. \tag{2}$$

We seek the solution in the form of the linear superposition of the deformed standing soliton solution of Eqs. (1), Ω_s, $\tilde{\Omega}_s$, and small perturbation $\delta\Omega$, $\delta\tilde{\Omega}$ satisfying the second of Eqs. (1.1):

$$\Omega(x,t) = \Omega_s(x) + \delta\Omega, \quad \tilde{\Omega}(x,t) = \tilde{\Omega}_s(x) + \delta\tilde{\Omega}, \tag{3}$$

Here,

$$\delta\Omega = i\varepsilon f_t(t)\varphi(x),$$
$$\delta\tilde{\Omega} = -i\varepsilon[f(t)\varphi_x(x) + \varphi_{1x}(x)]; \tag{4}$$

soliton components have the form $\Omega_s(x) = 0$ and $\tilde{\Omega}_s(x) = (4/\beta)\operatorname{sech}(\beta x)$, where $\beta \equiv \sqrt{2-\alpha}$ and α is the parameter of the soliton profile deformation such that $\alpha = 0$ corresponds to the exact soliton solution; ε is a small real parameter; and $f(t)$ and $\varphi(t)$ are real functions. The choice of perturbation $\delta\Omega$, $\delta\tilde{\Omega}$ in the form of imaginary additions (a phase shift of $\pi/2$ with respect to the soliton solution is generally necessary) ensures the elimination of cross terms $\int_{-\infty}^{\infty} dx\,(\delta\Omega\Omega_s^* + \delta\tilde{\Omega}\tilde{\Omega}_s^* + \text{c.c.}) = 0$ in the energy integral given in Eqs. (2). Thus, we exclude the interaction of the field components of the soliton with the internal mode but remain the possibility of the interaction of the internal mode with resonant oscillators. This interaction is described by Bloch equations (1.2), which have the following solutions for fields (3) and (4):

$$P(x,t) = -2\operatorname{sech}\beta x\tanh\beta x$$
$$+ i(-1 + 2\operatorname{sech}^2\beta x)\sin[\varepsilon(f\varphi + b\varphi_1)], \tag{5}$$
$$n(x,t) = (-1 + 2\operatorname{sech}^2\beta x)\cos[\varepsilon(f\varphi + b\varphi_1)],$$

where b is the integration constant determined from the initial conditions. Substituting Eqs. (3)–(5) into initial equations (1.1) and linearizing under the conditions ε, ω, $\alpha \ll 1$, we arrive at the expressions

$$f(t) = f_0\cos(\omega t + \phi_0), \quad \varphi_1(x) = \varphi_0\operatorname{sech}\beta x, \tag{6}$$

if $b = 1 - \alpha/2$. Here, ϕ_0 is the initial phase and the function amplitudes f_0 and φ_0 satisfy the conditions εf_0, $\varepsilon\varphi_0 \ll 1$. In what follows, $f_0 = 1$. The corresponding equation for the function $\varphi(x)$ has the form

$$\beta^{-2}\varphi_{xx} + \left[-1 + \frac{\omega^2 - \alpha}{2} + (2+\alpha)\sinh^2\beta x\right]\varphi = 0. \tag{7}$$

Using perturbation theory, it is easy to show that the eigenvalue problem specified by Eq. (7) has the finite localized solution

$$\varphi(x) = \operatorname{sech}\beta x - \frac{\alpha}{3}(1 + \ln\cosh\beta x)\operatorname{sech}\beta x, \tag{8}$$

if

$$\omega^2 = -\alpha/3. \tag{9}$$

Substituting Eqs. (6) and (8) into Eq. (4) and omitting the $\varepsilon\omega^2$ terms, we obtain the following expressions for the found internal modes:

$$\delta\Omega = -i\varepsilon\omega\sin\omega t\operatorname{sech}\beta x,$$
$$\delta\tilde{\Omega} = i\varepsilon\beta(\cos\omega t + \varphi_0)\operatorname{sech}\beta x\tanh\beta x. \tag{10}$$

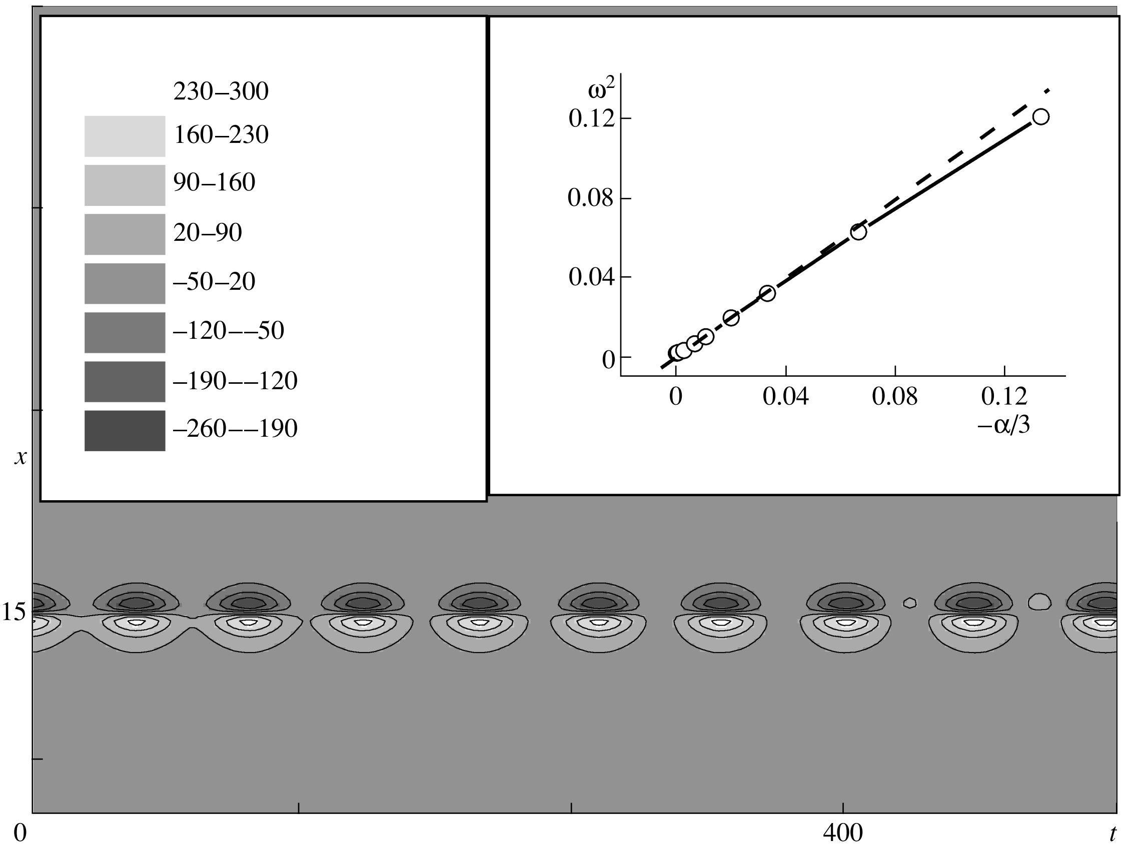

Fig. 1. Space–time dynamics of the difference field $\delta\tilde{\Omega}$ of the internal modes of the Bragg soliton in the presence of beatings of two modes with nonzero and zero frequencies. The inset shows the square of internal-mode oscillation frequency ω vs. the soliton-profile perturbation parameter α (dashed line) as calculated by Eq. (9) and (solid line) as obtained by numerically integrating Eqs. (1) with initial conditions (3), (5), and (10).

A low frequency of oscillations (9) is determined by the parameter of the soliton profile deformation $\alpha < 0$. The difference field $\delta\tilde{\Omega}$ given in Eqs. (10) is the superposition of two modes with nonzero and zero frequencies. The forms of these modes $\sim \operatorname{sech}\beta x \tanh\beta x$ are determined by functions φ_{1x} (6) and φ_x (8) and coincide with each other in the first approximation in the small parameter. This leads to effective beatings of these modes with the oscillating-mode frequency ω if $|\varphi_0| \approx 1$. Note that the presence of the zero-frequency internal mode for unperturbed soliton solutions [case $\omega = 0$ in Eq. (10)] is a characteristic feature of a number of nonlinear dynamic equations including the sine-Gordon equation [3]. Figure 1 shows the results of the numerical integration of Eqs. (1) with the initial conditions in the form of the analytical solutions given by Eqs. (3), (5), and (10) for the standing soliton with the internal mode. The absence of losses on the emission of continuous spectrum waves indicates that the solutions found for the internal mode are stable. The oscillation frequency (see the inset in Fig. 1) and the form of the internal mode agree well with analytical results (9) and (10). In addition, this figure shows that, owing to beatings, the energy of the internal-mode fields varies from zero to a certain maximum value. In this case, the energy of the system of excited two-level atoms changes by the corresponding value due to a change in the conversion $n(x, t)$ given by Eq. (5). In the case considered above for the Bragg soliton with zero velocity, the beatings of the internal modes do not change the soliton velocity. However, for the moving Bragg soliton, a change in the maximum inversion of atoms in beatings of the internal modes is an additional perturbation of the soliton and can lead to a considerable change in the pulse propagation velocity.

Let us generalize the solutions obtained for the perturbed standing soliton given by Eqs. (3), (5), and (10) to the Bragg soliton propagating with low velocity $v \ll \omega$. We assume that the form of the envelop of the envelop of the perturbed moving soliton coincides with the form of the exact solution [9], and the form of internal modes slightly differs from above expressions (10). Let the pulse center coordinate $\xi(t)$ and its velocity $v(t) = \xi_t(t)$ depend on time due to the perturbation of the soliton in the presence of beatings of the internal

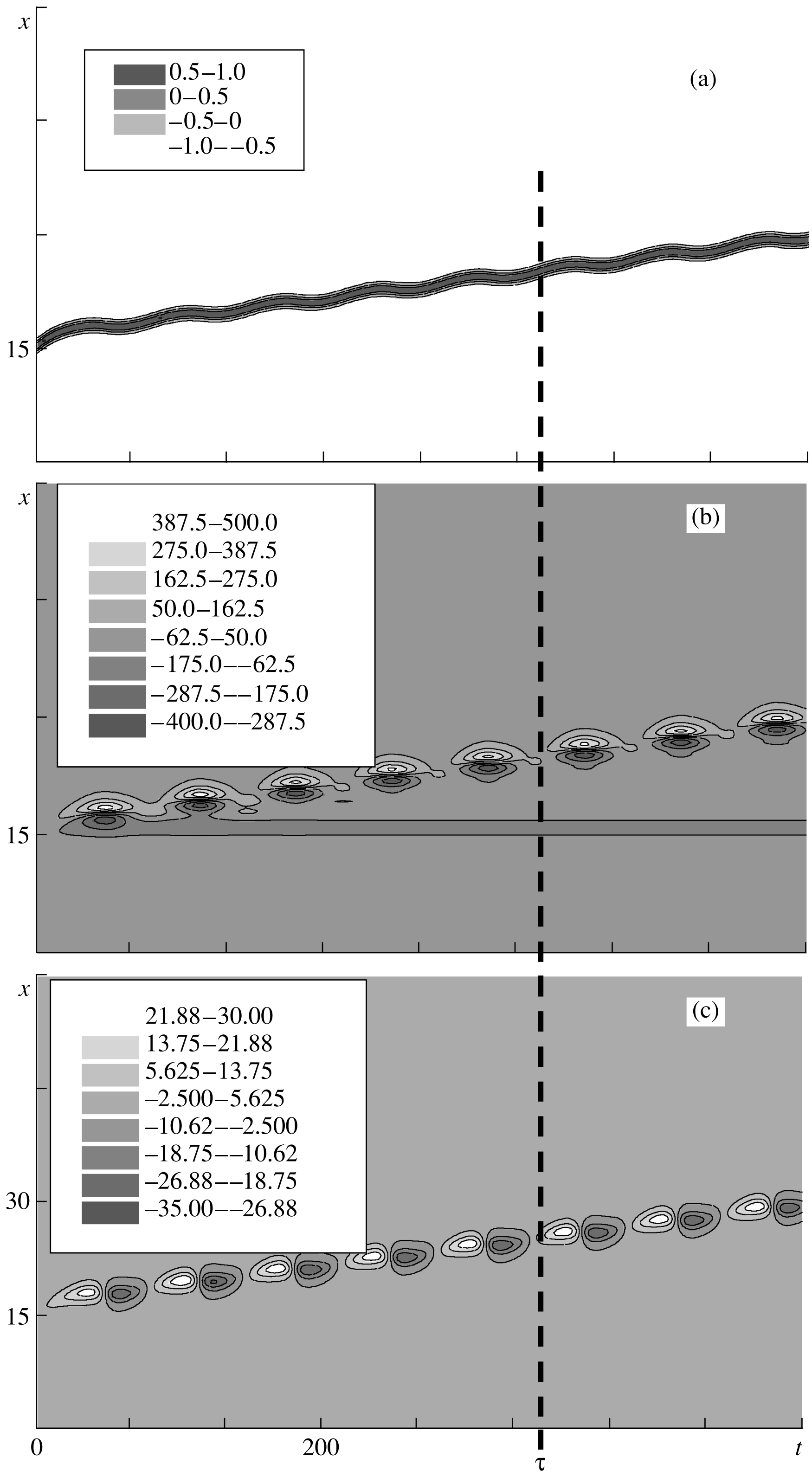

Fig. 2. Dynamics of the (a) inversion $n(x, t)$, (b) difference $\delta\tilde{\Omega}(x, t)$ (arb. units), and (c) total $\delta\Omega(x, t)$ (arb. units) internal-mode fields of the zoomeron-like pulse. The vertical dashed straight line corresponds to the time $t = \tau$ at which the amplitudes of internal-mode fields are equal to zero and the velocity of the pulse is maximal.

modes. Thus, the trial solution is chosen in the form

$$\Omega = \frac{4v(t)}{\beta\sqrt{1-v^2}}\operatorname{sech}\frac{\beta(x-\xi(t))}{\sqrt{1-v^2}} + i\varepsilon\omega\sin(\omega t)\operatorname{sech}\frac{\beta(x-\xi(t))}{\sqrt{1-v^2}},$$

$$\tilde{\Omega} = \frac{4}{\beta\sqrt{1-v^2}}\operatorname{sech}\frac{\beta(x-\xi(t))}{\sqrt{1-v^2}} - i\frac{\varepsilon\beta}{\sqrt{1-v^2}}(\cos\omega t+\varphi_0)\operatorname{sech}\frac{\beta(x-\xi(t))}{\sqrt{1-v^2}}\tanh\frac{\beta(x-\xi(0))}{\sqrt{1-v^2}},$$

$$n = \left(-1+2\operatorname{sech}^2\frac{\beta(x-\xi(t))}{\sqrt{1-v^2}}\right)\left(1-\frac{1}{2}\varepsilon^2(\cos\omega t+\varphi_0)^2\operatorname{sech}^2\frac{\beta(x-\xi(t))}{\sqrt{1-v^2}}\right), \quad (11)$$

$$P = -2\operatorname{sech}\frac{\beta(x-\xi(t))}{\sqrt{1-v^2}}\tanh\frac{\beta(x-\xi(t))}{\sqrt{1-v^2}} + i\varepsilon\left(-1+2\operatorname{sech}^2\frac{\beta(x-\xi(t))}{\sqrt{1-v^2}}\right)\times(\cos\omega t+\varphi_0)\operatorname{sech}\frac{\beta(x-\xi(t))}{\sqrt{1-v^2}}.$$

Substituting Eqs. (11) into the energy integral given in Eqs. (2), we arrive at the following expression for the pulse velocity:

$$v(t) = \frac{\varepsilon\omega\sqrt{\varphi_0}}{2}(1-\cos\omega t)^{1/2}. \quad (12)$$

As follows from Eqs. (11) and (12), the velocity and field amplitudes in the pulse, as well as the dipole moment and the inversion of atoms in the perturbed Bragg soliton, oscillate with the frequency of the internal mode of soliton (9), thus exhibiting the zoomeron-like dynamics of the pulse propagation.

In order to verify that the proposed trial solution given by Eq. (11) is sufficiently close to the exact solution, as well as to demonstrate the stability of such a zoomeron-like solution, we perform the direct numerical integration of Eqs. (1) taking analytical solution (11) as the initial conditions. As is seen in Fig. 2, the space–time dynamics of the inversion and internal-mode fields obtained in this integration correspond to analytical expressions (11). Similar results are also valid for the fields Ω_s and $\tilde{\Omega}_s$ of the soliton components of the solution and for the function of the dipole moment P. Topological charge (2) of the oscillating pulse obtained in numerical simulation satisfies the inequality $Q < 2\pi$, which corresponds to the analytical result with the substitution of solution (11) into Eq. (2): $Q = 2\pi + \alpha\pi$, where $\alpha < 0$. At the initial stage of the evolution of the solution, weak emission occurs (Fig. 2b), but the energy losses in this process are very small (about 0.05% of the pulse energy), which indicates that the trial zoomeron-like solution is close to the exact solution. The found zoomeron-like solution is quasistable, conserves stability for about one hundred oscillation periods, and elastically interacts with the soliton moving with velocity $v \geq 0.1$. The collision of two zoomeron-like pulses can be both elastic and inelastic, depending on the velocity and signs of the amplitudes of interacting pulses. The comparison of the plots in Fig. 2 provides a clear explanation of the cause of the appearance of oscillations in the zoomeron-like pulse. At $t = \tau$, when the pulse velocity is maximal (see Fig. 2a), the amplitudes of the internal-mode fields are equal to zero (see Figs. 2b and 2c). Further, the energy of internal modes increases due to the emission of the energy of excited medium atoms (the maximum inversion at the pulse center becomes less than unity in this case, see Fig. 3), the pulse stops and then the energy of the internal modes are absorbed by resonant atoms and the pulse is accelerated, again reaching the maximum velocity. Therefore, one can conclude that oscillations in the zoomeron-like pulse occur due to the beatings of the internal modes and to the energy exchange between the internal modes and resonant atoms, as follows from

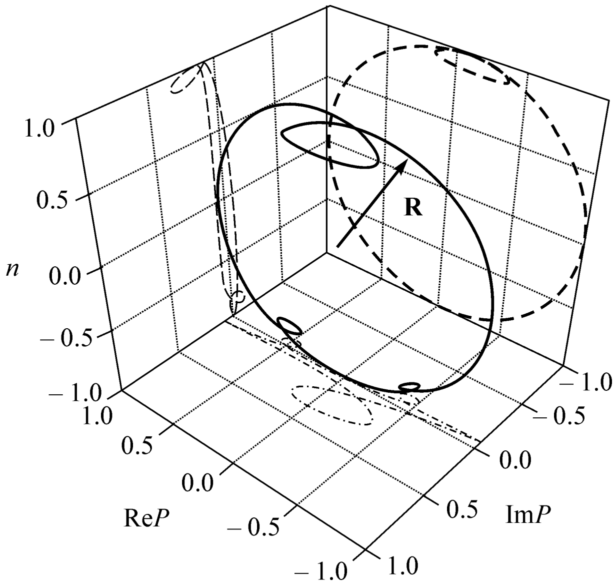

Fig. 3. (Solid line) Trajectory of the Bloch vector $\mathbf{R}(x = x_0; t) = \{\mathrm{Re}P; \mathrm{Im}P; n\}$ on a unit sphere at a certain point of the medium $x = x_0$ when the zoomeron-like pulse propagates. The dashed lines correspond to the projections of the shown trajectory on the coordinate planes. Each loop of the trajectory corresponds to one oscillation of the pulse.

analytical solutions (11) and (12). Moreover, the numerical simulation results confirm form (12) of the time dependence of the velocity of the zoomeron-like pulse and the linear frequency dependence of the maximum velocity. Thus, the solution given by Eqs. (11) and (12) for the zoomeron-like pulse well reproduces the dynamics of the oscillating pulse that is obtained in the direct numerical integration of two-wave Maxwell–Bloch equations (1).

In conclusion, we note that the two-wave Maxwell–Bloch equations, as follows from a number of the properties of their solutions, are incompletely integrable. It is difficult to expect that these equations have an exact zoomeron solution, which is an oscillating soliton of integrable nonlinear equations. For this reason, the approximate zoomeron-like solution described in this work is of interest as likely the first example of oscillating quasistable nonlinear solitary waves with nonzero average propagation velocity and a large amplitude of velocity oscillations, which appear in an actual physical problem, namely, in the problem concerning the propagation of laser pulses in the resonant Bragg structure.

I am grateful to A.I. Maĭmistov for stimulating discussions of the results. This work was supported by the Russian Foundation for Basic Research (project no. 04-02-16866).

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Translated by R. Tyapaev

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