JETP Letters, Vol. 74, No. 9, 2001, pp. 456–459. Translated from Pis'ma v Zhurnal Éksperimental'noĭ i Teoreticheskoĭ Fiziki, Vol. 74, No. 9, 2001, pp. 511–514.

Oscillating Gap 2π Pulse in Resonantly Absorbing Lattice

B. I. Mantsyzov* and R. A. Sil'nikov

Department of Physics, Moscow State University, Vorob'evy gory, Moscow, 119899 Russia

**e-mail: mants@genphys.phys.msu.su*

Received August 2, 2001; in final form, September 26, 2001

The interaction of a laser pulse with resonant Bragg lattice is studied theoretically for arbitrary initial conditions on the field, inverse population, and polarization of a medium. It is shown that the oscillating 2π self-induced transparency Bragg pulse can form if the Bragg conditions are exactly met. Various regimes are described for the oscillation dynamics of the gap 2π pulse. © *2001 MAIK "Nauka/Interperiodica".*

PACS numbers: 42.25.Fx; 42.50.Md; 42.65.Tg; 42.70.Qs

Over the past decade, the propagation dynamics of laser pulses in the photonic band gap structures, or photonic crystals [1], has been the subject of active studies, both theoretical [2–6] and experimental [7]. Because of the nonlinear interaction of radiation with such structures, the standard linear dispersion relations change qualitatively. As a result, Bragg solitons (BSs) [2–4], i.e., optical pulses with Bragg frequencies, can propagate in the linearly forbidden photonic gaps of the structures with different types of nonlinearity. Contrary to the solitons in continuous medium, the BSs are characterized by two propagation regimes: a regime with a constant velocity and an oscillating regime [5, 6] for which the pulse amplitude and the velocity magnitude and direction change periodically. A resonant oscillating 2π pulse was obtained earlier in [5] by solving numerically the problem on BS in a weakly distorted Bragg lattice. However, the physical nature of BS oscillations has not been revealed because of the lack of analytic solutions. It is shown in this work that the oscillating 2π pulse can arise when the Bragg conditions are exactly met. For the appropriate two-wave Maxwell–Bloch equations, the initial value problem reduces to a modified sine-Gordon equation (SGE). An analytic expression is obtained for the BS oscillation frequency, and the BS propagation regimes are described for different initial conditions.

Let us consider the coherent interaction of an intense laser radiation with a one-dimensional resonant Bragg lattice, whose structure consists of a set of periodically arranged thin layers containing two-level oscillators [2, 5]. This model closely corresponds to a real structure of periodically arranged quantum wells with resonance excitons in semiconductors [7]. The frequency of incident radiation coincides with the frequency of the two-level transition, and, for the Bragg condition to be met exactly, the structure period should be a multiple of the radiation half-wavelength. Under these conditions, the interaction of radiation with the structure is described by the two-wave Maxwell–Bloch equations [2] for real functions in the dimensionless variables $x = x'/c\tau_c$ and $t = t'/\tau_c$

$$\begin{gathered}\Omega_t^+ + \Omega_x^+ = P, \quad \Omega_t^- - \Omega_x^- = P,\\ P_t = n(\Omega^+ + \Omega^-), \quad n_t = -P(\Omega^+ + \Omega^-),\end{gathered} \tag{1}$$

where $\Omega^{\pm} = (2\tau_c\mu/\hbar)E^{\pm}$; $E^{\pm}$ are the smooth field-amplitude envelopes of the forward and backward Bloch waves; τ_c is the cooperative time; μ is the matrix element of the dipole transition moment; P and n are the polarization and density of inverse population, respectively; c is the speed of light; t' and x' are, respectively, the time and spatial coordinate along the normal to the resonance planes in the structure; and the subscripts x and t imply partial derivatives.

By using the solution $P = -\sin\theta$ to the Bloch equations, where the Bloch angle θ is determined from the condition $\theta_t = \Omega^+ + \Omega^-$, Eq. (1) can be rewritten as

$$\tilde{\Omega}_x + \Omega_t = -2\sin\theta, \quad \Omega_x + \tilde{\Omega}_t = 0, \tag{2}$$

where $\Omega \equiv \Omega^+ + \Omega^-$ and $\tilde{\Omega} \equiv \Omega^+ - \Omega^-$. From the second equation in Eqs. (2) it follows that

$$\tilde{\Omega}(x, t) = -\theta_x(x, t) + f(x). \tag{3}$$

Then the following equations are obtained for the Bloch angle from Eqs. (2):

$$\theta_{xx} - \theta_{tt} = 2\sin\theta + f_x(x). \tag{4}$$

This equation is a modified sine-Gordon equation, with the function $f(x)$ being determined by the initial condition in Eq. (3):

$$f(x) = \tilde{\Omega}(x, 0) + \theta_x(x, 0). \tag{5}$$

Therefore, if the fields and inverse population are absent in a medium at $t = 0$, i.e., if $\tilde{\Omega}(x, 0) = 0$ and $\theta(x, 0) = 0$, or if the stationary BS propagates in the structure and $\tilde{\Omega}(x, t) = -\theta_x(x, t)$ [2], then $f(x) = 0$ and Eq. (4) transforms into the exact SGE describing a nonoscillating gap 2π pulse. In the general case of $f(x) \neq 0$, the BS dynamics differs substantially from that described in [2]. The second term on the right-hand side of Eq. (4) corresponds to the interaction between a kink-solution of the exact SGE and a localized perturbation and gives rise to the oscillating regime of the 2π pulse.

The results of numerical integration of the original set of Eqs. (1) are presented in Fig. 1 for the nonzero initial conditions (5) $f(x) = f(0)\,\mathrm{sech}(\sqrt{2}x - x_0)$. For the low initial BS velocities and $f(0) < 0$ (Fig. 1, curve *a*), the oscillations of the BS amplitude and velocity are harmonic, the soliton shape is virtually identical with that of the solution to the exact SGE, and the oscillating BS is localized near the $f(x)$ function. The oscillation frequency depends on the $f(0)$ value. An increase in the initial velocity leads to a change in the form of oscillations (Fig. 1, curve *b*). It is demonstrated below that these oscillations obey the law of motion in the form of an elliptic sine. Finally, if the initial velocities are high, the soliton escapes from the $f(x)$ localization region and propagates as a free BS with a constant velocity and without oscillations (Fig. 1, curve *c*). As the amplitude of the function in initial condition (5) changes sign, i.e., if $f(0) > 0$, then the BS is repelled from the interaction region and also propagates with a constant velocity and without oscillations (Fig. 1, curve *d*).

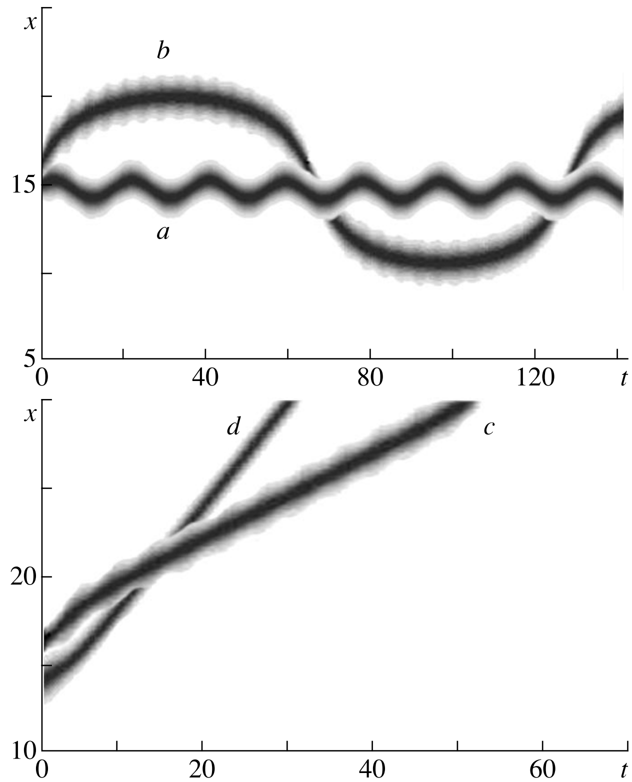

Fig. 1. Equivalue lines for the density $n(x, t)$ of on inverse population in a medium with the Bragg soliton propagating with different initial conditions. The black lines correspond to $n = 1$ and the white background corresponds to $n = -1$. At $t = 0$, the inversion and polarization are given by $n = -\cos(\theta)$ and $P = -\sin(\theta)$, respectively, where $\theta = 4\arctan[\exp(-\sqrt{2}x + x_0)]$ and $x_0/\sqrt{2}$ is the initial coordinate of the soliton center; the fields are $\Omega^{\pm} = \pm\Omega_0^{\pm}\mathrm{sech}(\sqrt{2}x - x_0)$, where $\Omega_0^+ = 1.41$ and $\Omega_0^- = 0.85$ for curve *a* corresponding to the soliton velocity $u = 0.2$; $\Omega_0^+ = 2.12$, $\Omega_0^- = 0.14$, and $u = 0.7$ for curve *b*; and $\Omega_0^+ = 2.21$, $\Omega_0^- = 0.06$, and $u = 0.76$ for curve *c*. In all cases, $f_0 < 0$ and $\sqrt{-f_0} = 0.75$. For curve *d*, $f_0 > 0$ and $\Omega_0^{\pm}$ are the same as for curve *a*.

To analyze this BS dynamics, we use a simple "energetic" method [8], which allows the law of motion to be determined for the soliton of modified SGE (4) in the case where its shape differs only slightly from the shape of the exact solution to the SGE. Let us substitute $\eta = \sqrt{2}x$, $\tau = \sqrt{2}t$, and $f' = f/\sqrt{2}$ and rewrite Eq. (4) in the standard form:

$$\theta_{\eta\eta} - \theta_{\tau\tau} = \sin\theta + f'_{\eta}(\eta). \tag{6}$$

The Lagrangian density function for Eq. (6) is

$$L = \frac{1}{2}\theta_\tau^2 - \frac{1}{2}(\theta_\eta - f')^2 - (1 - \cos\theta);$$

the corresponding Hamiltonian density is

$$H = \frac{1}{2}\theta_\tau^2 + \frac{1}{2}\theta_\eta^2 - f'\theta_\eta + \frac{1}{2}f'^2 + (1 - \cos\theta). \tag{7}$$

Note that the first four terms on the right-hand side of Eq. (7) are equal to the energy density $[(\Omega^+)^2 + (\Omega^-)^2]/2$ of the forward and backward waves in the structure.

Since the system is conservative, the total energy of the localized solutions is the integral of motion, $\frac{d}{d\tau}\int_{-\infty}^{\infty} H d\eta = 0$, so that from Eq. (7) it follows that

$$\frac{d}{d\tau}\int_{-\infty}^{\infty} d\eta\left(\frac{1}{2}\theta_\tau^2 + \frac{1}{2}\theta_\eta^2 + (1 - \cos\theta)\right) = \frac{d}{d\tau}\int_{-\infty}^{\infty} d\eta f'\theta_\eta. \tag{8}$$

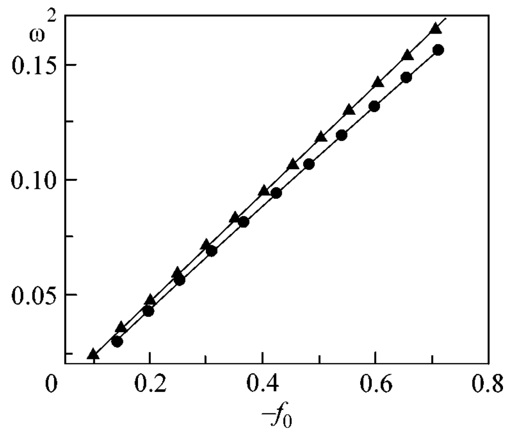

Fig. 2. The square ω^2 of the frequency of harmonic oscillations of the Bragg soliton (ω is in units of τ_c/c) vs. f_0; (▲) are obtained by numerical integration of the set of Eqs. (1) and (●) are calculated using analytic expression (13).

Making use of the fact that the shape of the oscillating soliton given by Eq. (6) differs only slightly from the solution to the exact SGE, one can write the desired solution for a 2π pulse propagating in the positive direction of the η axis as

$$\theta = 4\arctan\left[\exp\left(\frac{-\eta+\xi(\tau)}{\sqrt{1-u^2(\tau)}}\right)\right], \qquad (9)$$

where $u(\tau)$ is the time-dependent soliton velocity and $\xi(\tau) = \int_0^\tau u(\tau')d\tau'$ is the coordinate of the soliton center. The overlap integral on the right-hand side of Eq. (8) is the potential energy of interaction between kink (9) and the perturbation. Substituting Eq. (9) into Eq. (8) and taking into account that u^2 and $u_\tau \ll 1$, one obtains the following equation of motion for the coordinate of pulse center:

$$\xi_{\tau\tau} = -\frac{1}{4}\int_{-\infty}^{\infty}\operatorname{sech}(\eta-\xi)\tanh(\eta-\xi)f'(\eta)d\eta. \qquad (10)$$

Let $f'(\eta) = f_0\operatorname{sech}(\eta)$. The results of numerical integration of the original set of Eqs. (1) with the corresponding initial conditions are shown in Fig. 1. Then it follows from Eq. (10) that

$$\xi_{\tau\tau} = -\frac{f_0}{2}\frac{\sinh\xi - \xi\cosh\xi}{\sinh^2\xi}.$$

This equation can be recast as

$$\xi_{\tau\tau} = -U_\xi, \quad U = \frac{f_0}{2}\frac{\xi}{\sinh\xi}. \qquad (11)$$

Equation (11) describes the quasiparticle motion in the potential U in the field of potential force $-U_\xi$. Since the total "energy" of the particle $u^2/2 + U = \text{const}$, a finite motion is possible only in the attractive potential, i.e., only if $f_0 < 0$, and for a sufficiently low velocity $|u(\xi = 0)| < \sqrt{-f_0}$, i.e., at the bottom of the potential well. This agrees well with the results of numerical calculations (Fig. 1, curve *a*). An increase in the soliton velocity leads to its escape from the potential well (Fig. 1, curve *c*). If the initial conditions (5) are such that $f_0 > 0$, the interaction potential U is positive and the BS is repelled from the perturbation (Fig. 1, curve *d*).

The solution to Eq. (11) gives the law of BS motion $\xi(\tau)$ in the following integral form:

$$\int_0^\xi \frac{d\xi'}{\sqrt{\alpha - f_0\xi'/\sinh\xi'}} = \tau, \qquad (12)$$

where $\alpha = \xi_\tau^2(\xi = 0) + f_0$. By expanding the integrand in Eq. (12) in powers of ξ, one obtains, to second order in ξ, the following expression for the harmonic oscillations of BS with small deviations of the pulse center from equilibrium, $\xi \ll 1$, and $f_0 < 0$:

$$\xi = \xi_0\sin\omega\tau, \quad \omega^2 = -f_0/6. \qquad (13)$$

To the next order in ξ, the law of motion takes the form of an elliptic sine. The oscillation frequency of a gap 2π pulse, as obtained by the numerical integration of Eqs. (1) and calculated using Eq. (13), is shown in Fig. 2 as a function of f_0. One can see that the analytic formula agrees well with the numerically calculated dynamics of an oscillating 2π pulse.

Note in conclusion that the BS propagation dynamics is more complicated than the dynamics of optical solitons in continuum. This is caused by the interaction of BS with weak fields and medium excitation, which are localized within the Bragg band gap. The oscillating BS is a stable bound state of a high-energy pulse, close to the stationary soliton, and a low-energy perturbation. The latter needs not be necessarily static. The results obtained in this work can easily be extended to the "traveling" initial conditions through the transition to the moving frame of reference. In this case, the mean velocity of the oscillating BS will be nonzero. The oscillating 2π pulse can be observed experimentally, e.g., in the periodic structure of $In_{0.04}Ga_{0.96}As/GaAs$ quantum wells [7, 9], where the density of resonance excitons is 1.7×10^{12} cm^{-3}, the dipole transition moment $\mu = 9 \times 10^{-29}$ C m, the wavelength $\lambda = 830$ nm, and $\tau_c = 0.3$ ps. The corresponding pulse energy per unit area is equal to 1.3 μJ/cm^2 for the oscillating BS at a pulse duration of 0.34 ps.

This work was supported by the Russian Foundation for Basic Research, project no. 01-02-17314.

REFERENCES

1. *Photonic Band Gap Materials*, Ed. by C. M. Soukoulis (Kluwer, Dordrecht, 1996).
2. B. I. Mantsyzov and R. N. Kuz'min, Zh. Éksp. Teor. Fiz. **91**, 65 (1986) [Sov. Phys. JETP **64**, 37 (1986)].
3. W. Chen and D. L. Mills, Phys. Rev. Lett. **58**, 160 (1987).
4. C. Conti, S. Trillo, and G. Assanto, Phys. Rev. Lett. **78**, 2341 (1997).
5. B. I. Mantsyzov, Phys. Rev. A **51**, 4939 (1995).
6. F. De Rossi, C. Conti, and S. Trillo, Phys. Rev. Lett. **81**, 85 (1998).
7. J. P. Prineas, C. Ell, E. S. Lee, *et al.*, Phys. Rev. B **61**, 13863 (2000).
8. M. B. Fogel, S. E. Trullinger, A. R. Bishop, and J. A. Krumhansl, Phys. Rev. B **15**, 1578 (1977).
9. A. Schulzgen, R. Binder, M. E. Donovan, *et al.*, Phys. Rev. Lett. **82**, 2346 (1999).

Translated by V. Sakun